Optimal Filters for the Reconstruction and Discrimination of Reflectance Curves

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Abstract

Under the assumption that the interaction between objects and light is completely described by reflectance curves, we were able to determine filters to reconstruct and discriminate objects optimally. We found that in most cases the colour matching functions are not optimal and that sometimes better results are obtained with one well-chosen filter.[6]

Introduction

If images have to be processed to discriminate objects, we are mainly interested in the physical object characteristics. Image values are in most cases the result of the interaction of light with the object. In general several physical phenomena will take place[3], but to keep the problem tractable we suppose that the interaction is completely described by the reflectance curve.

In general, the reflectance curve consists of two contributions, i.e. specular reflection and object reflection[4,5]. Specular reflection is the reflection at the surface of the object, so its spectral power spectrum corresponds to the spectrum of the illuminant. For object reflection, the light penetrates into the object where it will be absorbed and partially re-emitted. After some time some light will leave the object. In contrast to surface reflection, the light due to object reflection will have a much more uniform distribution in all directions.

In general surface reflection can be avoided by making use of polarization filters, so we assume in the following sections that only object reflection is present.

Reflectance Curves

A reflectance curve indicates how much light is reflected by an object per unit wavelength. Reflectance curves are quite smooth, so they are still accurately described if the curves are sampled with a step of 20 nm. As distance measure between reflectance curves we make use of the normal Euclidean distance. As a result we will describe reflectance curves in a Euclidean N-dimensional vector space $<\mathbb{R}, \mathbb{R}^N, +>$.

To evaluate the spread of reflectance curves, the singular value decomposition (SVD) of two industrial colour sets was calculated, i.e. the IT8 chart developed by Agfa[1] and the Pantone catalog. The SVD of a set of M reflectance curves R_i is given by

$$S = \sum_{i=1}^{M} R_i R_i^t = \begin{pmatrix} & & \\ & U & \\ & & \end{pmatrix} \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_2 \end{pmatrix} \begin{pmatrix} & & \\ & U^t & \\ & & \end{pmatrix} \tag{1}$$

with S a symmetric matrix, U an orthogonal matrix, $\lambda_1, \ldots, \lambda_N$ the eigenvalues of S in decreasing order.

The columns in the matrix U are the basic reflectance curves. The first column corresponds to the reflectance curve that approximates the reflectance curves the best, the second column approximates the curves the second best, etc... Singular value decomposition of both sets indicates that most reflectance curves can be approximated by only a few basic reflectance curves.

Filters

A filter is a material through which some light passes. This behaviour is described by its transmission curve $F(\lambda)$. Because we will analyze reflectance curves with one or more filters, they will also be described in a Euclidean N-dimensional vector space $<\mathbb{R}, \mathbb{R}^N, +>$.

The response r of a linear recording device can be modelled as

$$r = \int R(\lambda)\, I(\lambda)\, F(\lambda)\, G_c(\lambda)\, d\lambda \tag{2}$$

with $I(\lambda)$ the power spectrum of the illuminant, $R(\lambda)$ the reflectance curve of the object, $F(\lambda)$ the filter before the linear recording device, $G_c(\lambda)$ the sensitivity of the linear recording device.

If the illuminant, the filter and the sensitivity of the camera are taken together as the global filter, the response of the camera is the scalar product of the reflectance curve and the global filter. In the following sections optimal global filters will be constructed. They will curtly be referred to as filters. The response r will be called the filter value or the image value.

Reconstruction of Reflectance Curves

If an image is recorded with K independent filters, the reflectance curve can be reconstructed into this K dimensional space. The best choice for K filters are these for which the transmission curve corresponds to the first K basis reflectance curves. However, apart from the first calculated filter, all the other determined filters have both negative and positive values, whereas real filters can only have positive values. But because the first filter is strict positive (Perron-Frobenius theorem [2,7]), combinations of this filter with the k^{th} filter can be made such that all filter values are positive. These filters are referred to as the progressive optimal filters because the best K+l filters can be obtained by extending the best K filters with one filter.

Discrimination of Reflectance Curves

In a similar way as for the reconstruction of reflectance curves, filters can be designed to reconstruct differences between reflectance curves. The first three progressive optimal filters for the five reflectance curves in the upper left image of Figure 1 are represented in the upper right image.

For image processing purposes, we are sometimes interested in discriminating objects as good as possible with one filter. This can be done in several ways. We calculated the filter:

1. to maximize the sum of the squared differences between all filter values. This filter is in most cases a delta filter. For the reflectance curves in the up per left image of Figure 1 this filter is a delta filter at 640 nm.
2. to spread the filter values equidistantly and maximally (equidistant filter). The maximum difference between the filter values for the curves of Figure 1 is 13.5 (on a scale of 255); the corresponding filter is represented in the lower left part of Figure 1.
3. to maximize the minimum difference between filter values (optimal discriminating filter). The optimal discriminating filter is given in the lower right part of Figure 1. The maximum minimum difference between filter values is 20.20 (on a scale of 255).

Colour Matching Functions

To analyze how good M reflectance curves can be approximated with K filters, the following criteria are defined.

$$C_1 = \frac{\sum_{j=1}^{K} \sum_{i=1}^{M} < R_i, O_j >^2}{\sum_{j=1}^{N} \lambda_j} \tag{3}$$

$$C_2 = \frac{\sum_{j=1}^{K} \sum_{i=1}^{M} < R_i, O_j >^2}{\sum_{j=1}^{K} \lambda_j} \tag{4}$$

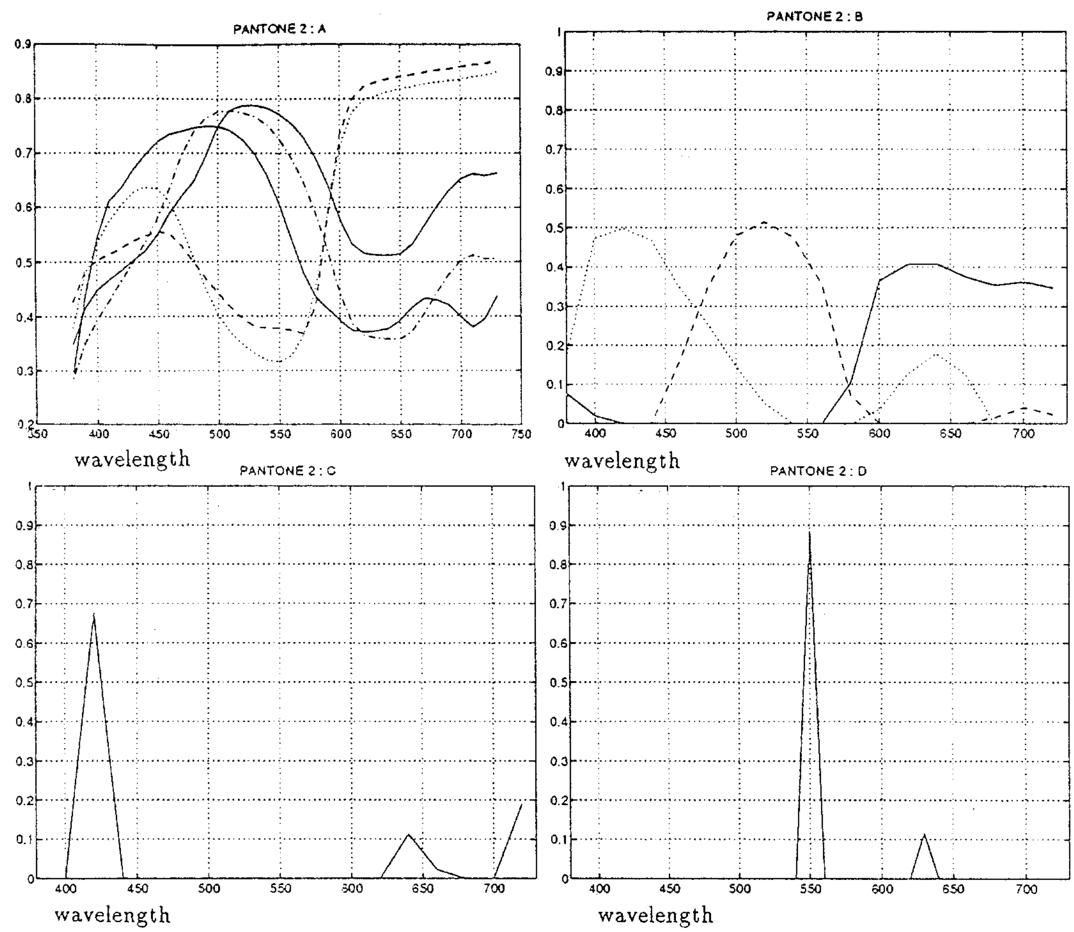

Figure 1. A) 5 reflectance curves of the Pantone catalog; B) progressive optimal filters for reconstruction; C) equidistant filter; D) optimal discriminating filter

Table 1. Values of the criteria C_1 and C_2 for different sets of reflectance curves.

		Colour matching functions				Optimal filters			
K		IT 8.1	IT 8.2	Pantone 1	Pantone 2	IT 8.1	IT 8.2	Pantone 1	Pantone 2
1	C_1	0.4235	0.8574	0.4729	0.5700	0.4601	0.8769	0.7960	0.8319
	C_2	0.7639	0.8629	0.7980	0.6985	0.7358	0.8829	0.8040	0.8402
3	C_1	0.8197	0.8629	0.6650	0.6981	0.9951	0.9993	0.9998	0.9070
	C_2	0.8200	0.8629	0.6841	0.7208	0.9956	0.9995	0.9999	0.9070

with O_j an orthogonal basis of the K filters and λ_j the eigenvalues of the matrix S, $< a, b >$ the scalar product between the vectors a and b.

The measure C_1 indicates how good the M reflectance curves can be absolutely approximated with the K filters. This measure can have values from 0 to 1 with $C_1 = 0$ if the K filters are perpendicular to the reflectance curves and $C_1 = 1$ if all the curves can be written as a sum of the K filters. The measure C_2 on the other hand, indicates how good the reflectance curves can be approximated relative to the optimal choice.

In Table 1 four sets of five colour curves are taken, two sets out of the IT8 chart and two of the Pantone catalog. For these sets the criteria C_1 and C_2 are calculated for the best K filters in the space with the colour matching functions (CMF) as base. To evaluate how good the CMF are at discriminating reflectance curves, these criteria are also determined for the progressive optimal filters for reconstruction of the reflectance curves. Table 1 indicates that in three cases out of four better results are obtained by using one optimal filter than with the colour matching functions.

Conclusions

If the interaction of objects with light is completely described with the object reflectance curve, this curve can be characterized for physical inspection tasks. With linear recording devices, the image values can be considered as the projection of the reflectance curve on a given vector, the global filter. If several filters are used, it is possible to reconstruct the reflectance curve in the space based on the corresponding global filters.

By making use of linear algebra, optimal filters can be designed to reconstruct reflectance curves or differences between them. On the other hand, in inspection tasks one tries to discriminate objects by making use of only one image value per pixel. We have also indicated that it is possible to construct an optimal filter to spread the filter values according to several criteria. If the performance of the optimal filters is compared with the colour matching functions, generally better results are obtained with well-chosen filters.

By making use of these techniques we are able to construct optimal filters and in this way to optimize the recording system. On the other hand, this technique indicates the images have to be processed, i.e. how the components have to be taken together if several filters are used and/or the choice of optimal threshold values in image segmentation procedures.

References

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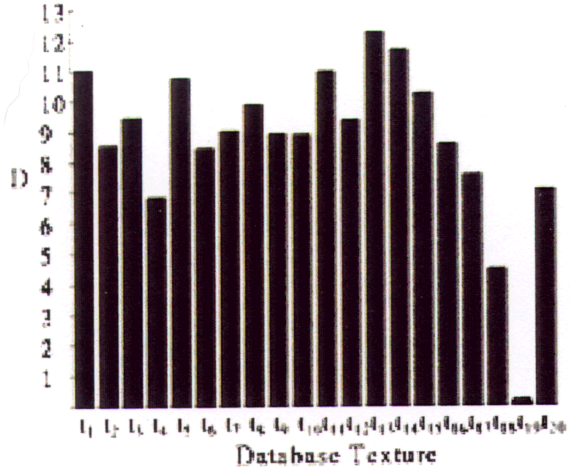

Figure 1. Computed D for t_{19}

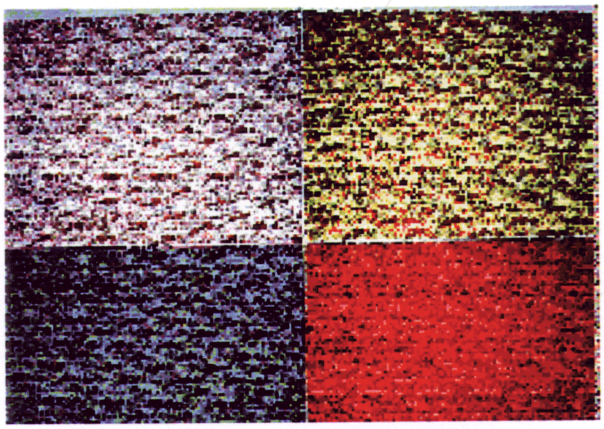

Figure 4. Patterned Cloth (texture 3)

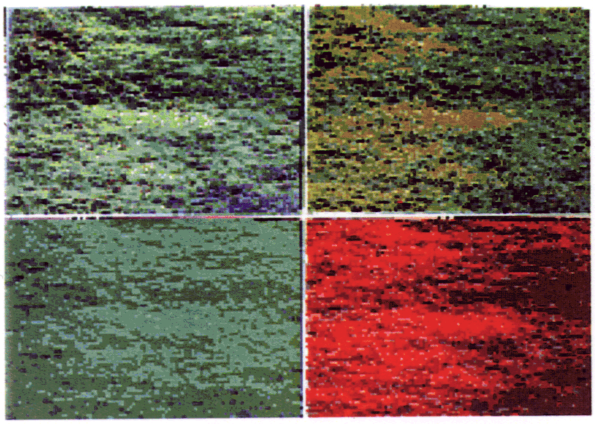

Figure 2. Tree (texture 19)

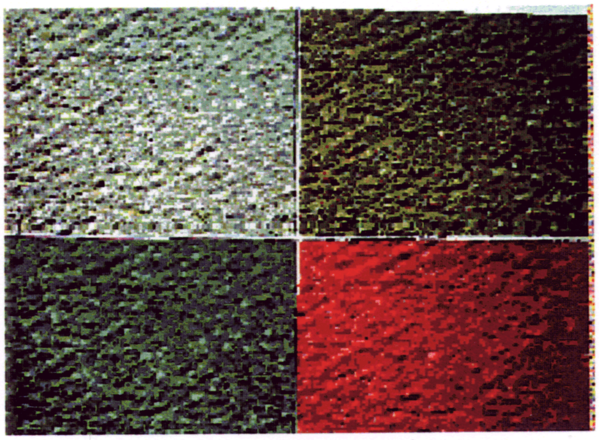

Figure 5. Sand (texture 17)

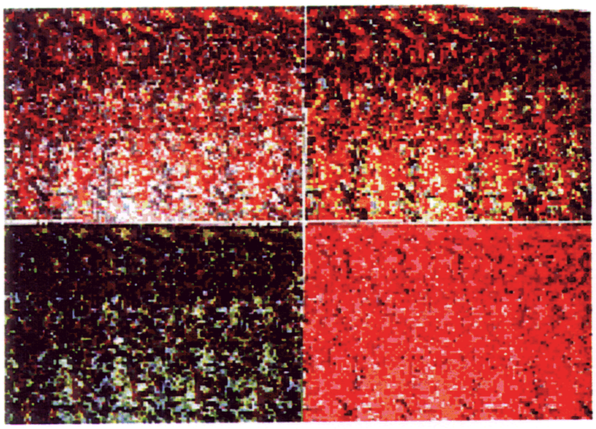

Figure 3. Carpet (texture 5)

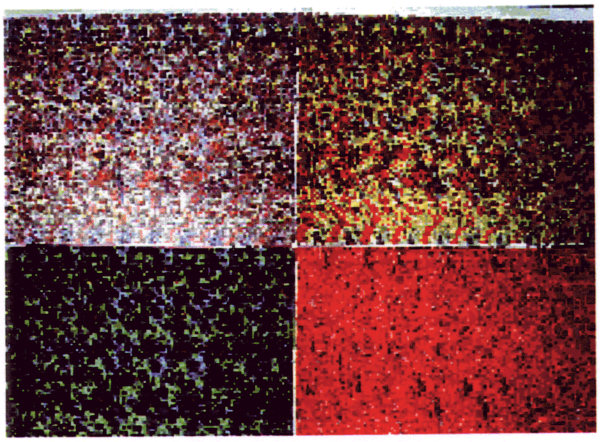

Figure 6. Carpet (texture 16)